

NPS55-83-028

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



WHOLESALE PROVISIONING MODELS:  
MODEL OPTIMIZATION

by

G. T. Howard

October 1983

Final Report for Period April 1983 - September 1983

Approved for public release; distribution unlimited.

Prepared for:

Commanding Officer  
Fleet Material Support Office  
Mechanicsburg, PA 17055

FedDocs  
D 208.14/2  
NPS-55-83-028

NAVAL POSTGRADUATE SCHOOL  
Monterey, California

R. H. Shumaker  
Superintendent

David A. Schrady  
Provost

The work reported herein was supported with funds provided by the Navy Fleet Material Support Office, Mechanicsburg, Pennsylvania.

Reproduction of all or part of this report is authorized.

This report was prepared by:

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

LIBRARY  
GRADUATE SCHOOL  
MONTEREY CA 93943-5101

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS55-83-028	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) WHOLESALE PROVISIONING MODELS: MODEL OPTIMIZATION		5. TYPE OF REPORT & PERIOD COVERED FINAL REPORT 1 April 83 - September 83
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) G. T. Howard		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS N0036782PON3901
11. CONTROLLING OFFICE NAME AND ADDRESS Commanding Officer Fleet Material Support Office Mechanicsburg, PA 17055		12. REPORT DATE October 1983
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Provisioning Availability Inventory Models Mean Supply Response Time Dynamic Programming		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Wholesale provisioning models are considered. The performance measures used are: supply material availability, mean supply response time, and availability. These measures are optimized subject to a budget constraint. The performance constrained budget minimization problems are also solved for the optimal allocation. The basic method used is dynamic programming.		



## Abstract

Wholesale provisioning models are considered. The performance measures used are: supply material availability, mean supply response time, and availability. These measures are optimized subject to a budget constraint. The performance constrained budget minimization problems are also solved for the optimal allocation. The basic method used is dynamic programming.



## I. Introduction

### A. Overview

The purpose of this report is to discuss solution methods for several problems arising in inventory provisioning. Two related types of problems are considered:

- 1) optimization of a performance measure subject to a budget constraint
- 2) minimization of cost subject to a constraint on performance.

The performance measures considered are Supply Material Availability (SMA), Mean Supply Response Time (MSRT), and "Pseudo-Availability" (PA). The basic approach to these problems is dynamic programming.

First the problems are formulated and solved using one recursive technique. Then a more efficient recursion is presented and discussed in the context of maximizing MSRT subject to a budget constraint.

These same budget constrained problems are formulated, discussed and solved using a marginal analysis approach in [2] and [3]. That method, although fast, does not guarantee that optimal solutions are obtained. This report provides a method of obtaining optimal solutions and thus provides a means of evaluating heuristic methods. In addition, this report shows how the performance constrained budget minimization problems can be solved directly. By contrast, reference [2] addresses this problem using generalized Lagrange Multipliers or by solving the budget constrained performance problem repeatedly for various budget levels.

The second recursion presented here is considerably faster than the first and it also guarantees an optimal solution in some cases. It is competitive in speed with the marginal analysis method for small and medium-sized

problems, but it is inefficient for problems with a large number of items or large budget values. Its virtue lies in its ability to get exact solutions to medium-sized problems.

#### B. Problem Formulations, budget constrained

This report considers three specific budget constrained optimization problems arising in inventory provisioning. These problems are formulated and discussed in detail elsewhere [2] and will be stated here without extensive explanation.

Following [2] we let

$n$  = the total number of items considered for provisioning

$C_i$  = the unit cost of item  $i$

$E_i$  = the essentiality code for item type  $i$

$\lambda_i$  = the demand rate for item  $i$

$T_i$  = the procurement leadtime for item  $i$

$S_i$  = the number of items of type  $i$  provided (the decision variables)

$Z_i(S_i)$  = the performance measure for item  $i$  when  $S_i$  units are stocked

$D_i(S_i) = Z_i(S_{i+1}) - Z_i(S_i)$

$p_i(x_i)$  = probability that demand for item  $i$  is  $x_i$  during the provisioning interval

$P_i(x_i)$  = cumulative probability of  $x_i$  or fewer demands during a provisioning interval

$MTTR_i$  = mean time to repair or replace item  $i$

$MTBF_i$  = mean time between failures =  $1/\lambda_i$

$MSRT_i(S_i)$  = mean supply response time when  $S_i$  units are stocked.



The three budget constrained problems considered are:

a1) maximize Supply Material Availability (SMA), defined as

$$SMA(S_1, \dots, S_n) = \frac{\sum_{i=1}^n E_i \lambda_i T_i Z_i^{(1)}(S_i)}{\sum_{i=1}^n E_i \lambda_i T_i}$$

where

$$Z_i^{(1)}(S_i) = (1 - p_i(S_i)) + (S_i - \lambda_i T_i)(1 - P_i(S_i))/\lambda_i T_i .$$

b1) minimize Mean Supply Response Time (MSRT), defined as

$$MSRT(S_1, \dots, S_n) = \frac{\sum_{i=1}^n E_i \lambda_i T_i Z_i^{(2)}(S_i)}{\sum_{i=1}^n E_i \lambda_i T_i}$$

where

$$\begin{aligned} Z_i^{(2)}(S_i) = & (1 - P_i(S_i))(T_i/2 - S_i/\lambda_i + \frac{S_i(S_i+1)}{2\lambda_i^2 T_i}) \\ & + p_i(S_i)(\lambda_i T_i - S_i)/2\lambda_i . \end{aligned}$$

c1) maximize Pseudo-Availability (PA), defined as

$$PA(S_1, \dots, S_n) = \prod_{i=1}^n Z_i^{(3)}(S_i)$$

where

$$Z_i^{(3)}(S_i) = MTBF_i / (MTBF_i + MTTR_i + Z_i^{(2)}(S_i)) .$$

In each of the budget constrained problems above the objective is to be optimized by selection of  $S_1, \dots, S_n$  subject to the constraints

$$\sum_{i=1}^n C_i S_i \leq B, \quad S_i \geq 0 \text{ integer}$$

where  $B$  is the specified budget level.

### C. Problem Formulations, Performance Constrained

In addition to the three problems just stated, we consider three related problems in which the cost is to be minimized subject to a constraint on performance.

$$\begin{aligned} \text{a2) } \min \quad & \sum_{i=1}^n C_i S_i \\ \text{s.t.} \quad & \text{SMA}(S_1, \dots, S_n) \geq \text{SMA} \\ & S_i \geq 0 \text{ integer.} \end{aligned}$$

$$\begin{aligned} \text{b2) } \min \quad & \sum_{i=1}^n C_i S_i \\ \text{s.t.} \quad & \text{MSRT}(S_1, \dots, S_n) \leq \text{MSRT} \\ & S_i \geq 0 \text{ integer.} \end{aligned}$$

$$\begin{aligned} \text{c2) } \min \quad & \sum_{i=1}^n C_i S_i \\ \text{s.t.} \quad & \text{PA}(S_1, \dots, S_n) \geq \text{PA} \\ & S_i \geq 0 \text{ integer.} \end{aligned}$$

## II. Solution Method and Examples

### A. Dynamic Programming Approach

The computer program used to solve these problems is DP4, a general purpose program for performing dynamic programming tabular computations. Here we will describe the general nature of that program and those elements required to tailor it for use in the problems considered in this report.

The DP4 program deals with a problem consisting of  $n$  related stages each of which is characterized as shown in figure 1.

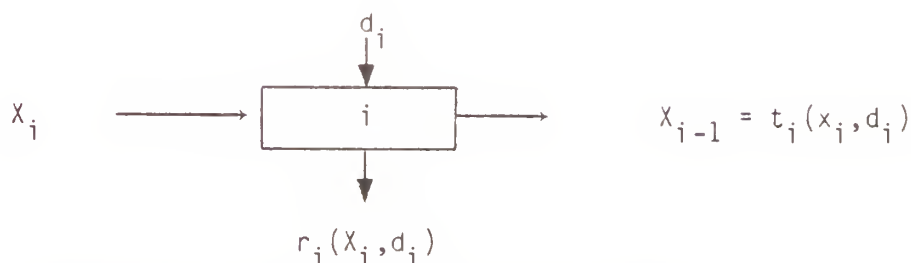


Figure 1. A single-stage decision problem.

In figure 1

$x_i$  is the "state" variable

$d_i$  is the decision variable

$r_i$  is the stage return function

$t_i$  is the stage transformation function.

In the overall problem consisting of  $n$  stages the output state variable from stage  $i$ , namely  $x_{i-1}$ , is the input to stage  $i-1$ . Thus, the  $n$  stage problem can be pictured as in figure 2.

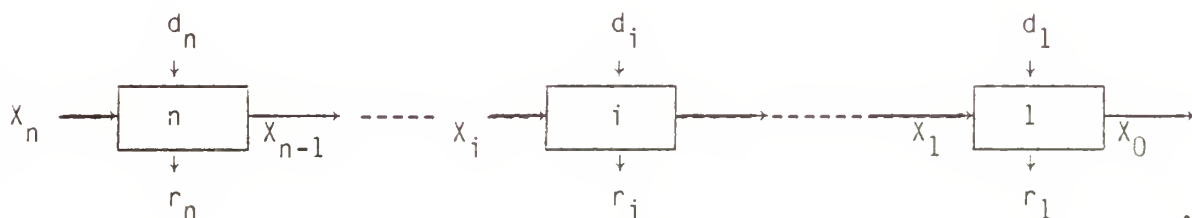


Figure 2.  $n$ -stage decision problem.

The state variable  $X_i$  can easily be understood in the context of the budget constrained problems as the amount of resource (money) remaining to be allocated to stages  $i, i-1, \dots, 1$ . The stages, of course, correspond to the items in the inventory problems.

At each stage  $i$  a decision  $d_i$  must be made. The decision has two effects. First, it yields a return  $r_i$ , the performance measure associated with the current item. Second, it yields a value of  $X_{i-1}$  which serves as the input to the remainder of the decision process. The decision  $d_i$  must be made with consideration both for the immediate return  $r_i$  and the future state  $X_{i-1}$ . The overall problem is to make the series of decisions  $d_n, \dots, d_1$  to optimize some function of the individual stage returns.

In the problem (a1), where  $S_i$  is the decision variable, we can let the return functions be

$$r_i(X_i, S_i) = E_i \lambda_i T_i Z_i^{(1)}(S_i) / \sum_{i=1}^n E_i \lambda_i T_i \quad i = 1, \dots, n$$

and the stage transformation functions be

$$X_n = B$$

$$X_{i-1} = t_i(X_i, S_i) = X_i - C_i S_i \quad i = 1, \dots, n.$$

The overall return function is the sum of the individual return functions. Namely,

$$SMA(S_1, \dots, S_n) = \sum_{i=1}^n r_i(X_i, S_i).$$

The object remains to select  $S_1, \dots, S_n$  to optimize this return.

We let  $f_i(X_i)$  = the optimal total return from stages  $i, i-1, \dots, 1$

given that we enter stage  $i$  with state variable  $X_i$ .

Then we can write the recursive equations for this optimization as

$$f_i(X_i) = \max_{S_i} \{r_i(X_i, S_i) + f_{i-1}(X_{i-1})\}$$

$$\text{s.t.} \quad X_{i-1} = X_i - C_i S_i$$

$$\text{and} \quad 0 \leq S_i \leq X_i/C_i$$

$$\text{and} \quad S_i = \text{integer}$$

for  $i = 2, \dots, n$ .

The equation at stage 1 is

$$f_1(X_1) = \max_{S_1} r_1(X_1, S_1)$$

$$\text{s.t.} \quad 0 \leq S_1 \leq X_1/C_1$$

$$\text{and} \quad S_1 \text{ integer.}$$

The program DP4 performs this optimization provided the user supplies the following subroutines and data.

Required subroutines

1. STGRET - this subroutine defines the function  $r_i(X_i, d_i)$
2. TRANFM - defines the stage transformation function  $t_i(X_i, d_i)$
3. DLIMIT - defines the range of decision values  $d_i$  which can be considered for the particular value of  $X_i$  under consideration
4. STORE - allows the input of constants to be used in the other subroutines.

## Required Data

1.  $n$  - the number of stages
2. For each stage  $i$ 
  - XLOW - the lowest value of  $X_i$  to consider
  - XHIGH - the highest value of  $X_i$  to consider
  - DELX - the increment for  $X_i$
  - XMODE - tells whether to maximize or minimize
  - XSTAGE - tells how this stage return relates to lower numbered stage returns (sum, product).

The methodology is essentially the same for the performance constrained problems. There the return functions  $r_i(X_i, d_i) = c_i d_i$ . The state variable  $X_i$  is interpreted as the portion of the performance measure to be attributed to stages  $1, \dots, i$ . The stage transformation functions in problem (a2) and (b2) are

$$X_{i-1} = X_i - Z_i(S_i) .$$

In problem (c2) the stage transformation is

$$X_{i-1} = X_i / Z^{(3)}(S_i) .$$

The program DP4 and the subroutines are shown in Appendix A. The subroutines are written to solve any of the problems (a1), (b1), (c1) or (a2), (b2), (c2). Thus they involve complications not needed for solving just one of these problems.

## B. Examples

Several example problems were solved to illustrate the approach discussed here. All of the problems involved  $n = 10$  items and all used the data shown in table 1.

Data

n	$\lambda$	Time	Cost	MTTR	E
1	5.0	1.0	1.0	.0137	1.0
2	2.0	1.0	2.0	.0274	1.0
3	3.0	1.0	5.0	.0137	1.0
4	5.0	1.0	10.0	.0822	1.0
5	10.0	1.0	20.0	.0274	1.0
6	25.0	1.0	5.0	.0027	1.0
7	1.0	1.0	1.0	.0054	1.0
8	1.0	1.0	100.0	.0411	3.0
9	0.5	1.0	50.0	.0082	1.0
10	2.0	1.0	10.0	.1370	3.0

Table 1: Data for Examples

1. The budget constrained problems.

Tables 2, 3, and 4 summarize the solutions for the example problems (a1), (b1), and (c1). These are the budget constrained problems.

B = Budget =	300	295	290	285	280
max SMA =	.750654	.746466	.741160	.736563	.732964
decision $S_1 =$	4	4	4	4	4
$S_2 =$	4	4	4	4	4
$S_3 =$	4	4	4	4	4
$S_4 =$	5	5	5	4	5
$S_5 =$	2	2	2	2	1
$S_6 =$	29	28	27	28	29
$S_7 =$	3	3	3	3	3
$S_8 =$	0	0	0	0	0
$S_9 =$	0	0	0	0	0
$S_{10} =$	3	3	3	3	3

Table 2: Solutions to example problem (a1)



B =		300	295	290	285	280
min MSRT =		.0838231	.0859625	.0878466	.0900560	.0927486
decision	$S_1$	2	2	2	2	2
	$S_2$	3	3	3	3	3
	$S_3$	3	4	3	3	3
	$S_4$	4	4	4	4	4
	$S_5$	5	4	4	4	4
	$S_6$	21	23	23	22	21
	$S_7$	2	2	2	2	2
	$S_8$	0	0	0	0	0
	$S_9$	0	0	0	0	0
	$S_{10}$	3	3	3	3	3

Table 3. Solutions to example problem (b1)

B = max PA =		300 .0403727	295 .0387421	290 .037005	285 .0355446	280 .0341911
decision	$S_1$	3	3	3	3	3
	$S_2$	4	4	4	4	4
	$S_3$	5	4	5	5	5
	$S_4$	5	5	5	5	5
	$S_5$	3	3	2	2	2
	$S_6$	26	26	26	27	26
	$S_7$	4	4	4	4	4
	$S_8$	0	0	0	0	0
	$S_9$	0	0	0	0	0
	$S_{10}$	2	2	3	2	2

Table 4. Solutions to example problem (c1)

2. The performance constrained problems.

The related performance constrained problems were also solved for illustration. For example, the problem (a2) was solved using the data from Table 1 with the restriction that  $SMA \geq .732964$ . The solution to that problem is the same as the solution shown in the last column of Table 2 since the value .732964 is the (largest) value of SMA corresponding to a budget of 280.

### III. Modifications to Basic Method

#### A. An Alternative Recursion

An alternative and more efficient approach is available for the budget constrained problems.

The approach is based on a different recursion from that used in Section 2 and is very similar to an approach used to solve the "cargo loading problem". See for example Dreyfus [1].

The cargo loading problem, stated as a maximization, is:

$$\begin{aligned} \max \quad & \sum_{j=1}^N v_j d_j \\ \text{s.t.} \quad & \sum_j c_j d_j \leq B \\ & d_j \geq 0 \text{ integer.} \end{aligned}$$

Although many methods are available for solving this problem, the one of interest to us is based on the following recursion

$$f(b) = \max_{j \in \{1, \dots, N\}} \{v_j + f(b - c_j)\}$$

where  $f(b)$  is the optimal total return that can be obtained when a budget of  $b$  is available.

To illustrate this method consider the data in Table 5.

$i$	1	2	3 = N	
$v_i$	1	4	6	$B = 10$
$c_i$	1	3	4	

Table 5. Data for example using alternative recursion

The computation proceeds with increasing values of  $b$  until  $b = B$  is reached. The process can be viewed as shown in figure 3 where a template

representing the available items is placed over the budget value of current interest. The template points back to previously determined optimal solution. Each of these previous solution is considered for updating by including one more item of the type inducted by the template. The updated solutions are compared and the best selected as the solution for the current value of  $b$ . The illustration shows the template at the budget value of 7. The optimal solutions for  $b = 0, 1, \dots, 6$  have already been computed. The comparison at  $B = 7$  is among the solution at 6 with an additional item 1 for a total return of 9, the solution at 4 with an additional item of type 2 for a total return of 10, and the solution at 3 with an additional item of type 3 for a return of 10. Either of the last two is chosen and recorded as an optimal solution at 7.

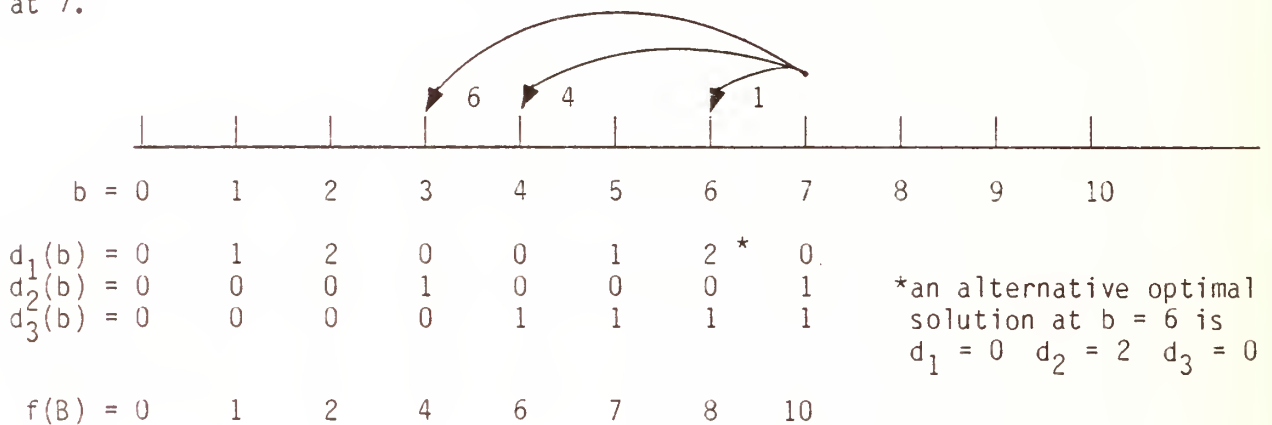


Figure 3: Illustration of solution method using the alternative recursion.

A very simple modification of this procedure can be used to solve the problems of the type discussed in this report. We consider

$$\begin{aligned}
 &\max \sum_{j=1}^n r_j(d_j) \\
 &\text{s.t.} \quad \sum_{j=1}^n c_j \cdot d_j \leq B \\
 &\quad \quad d_j \geq 0 \text{ integer}
 \end{aligned}$$

where the return functions  $r_j(d_j)$  are concave. The procedure below is not guaranteed to give the optimal solution for all  $r_j(d_j)$  but is guaranteed if the  $r_j(d_j)$  are points on a concave function.

In this case we will represent the return functions  $r_j(d_j)$  as the sum of the marginal values of additional items of type  $j$ .

$$r_j(d) = + \sum_{i=1}^d m_j(i)$$

Thus  $r_j(2) = m_j(0) + m_j(1) + m_j(2)$ . These marginal values form a sequence with the properties that

$$m_j(i) > 0 \quad i$$

and

$$m_j(i) > m_j(k) \quad i < k .$$

The same algorithm as before was applied with the modification that the value term  $v_j$ , which was formerly constant, is replaced by  $m_j(i)$  for the appropriate value of  $i$ . The program which implements this algorithm was called RECUR.

It should be noted that the discussion above treats the constraint as an inequality, but the function  $f(b)$  in this section is computed for the constraint

$$\sum_{j=1}^N c_j x_j = b .$$

For this reason we may have in a maximization problem

$$f(b_1) > f(b_2)$$

although  $b_1 < b_2$ . That is, if we require the equality to be met exactly, it is not necessarily true that a larger budget is better. The program prints the values of  $f(b)$  for several values of  $b$  so the optimal value of  $b \leq B$  can be found visually.

## B. Minimum Orders

### 1) RECMOD

A modification was made to the program RECUR to permit the user to specify minimum packaging quantities of each item. That is, item  $i$  is assumed to be packaged with  $q_i$  items per package. The provisioning can select only whole packages of each item. This modification resulted in the program RECMOD given in the Appendix.

### 2) Example

To illustrate the RECMOD program, consider the example problem (a2) solved previously. The optimal solution for budget  $B = 300$  is repeated in Table 2 for the case in which all  $q_i = 1$ .

n	1	2	3	4	5	6	7	8	9	10	Objective
$q_i$	1	1	1	1	1	1	1	1	1	1	
$d_i$	2	3	3	4	5	21	2	0	0	3	.0838231
$q_i$	2	1	1	5	1	1	1	3	1	1	
$d_i$	2	3	3	5	4	23	2	0	0	3	.0847410

Table 6. Solution to Example using RECMOD.

The optimal solution is also shown for a modified problem in which not all  $q_i$  are equal to 1.

### C. Discussion

The modified recursion just discussed has been implemented for the problem (a2) which is to minimize MSRT subject to a budget constraint. The method can also be applied to the other budget constrained problems, but this has not yet been done. All that is required is to modify the program to compute SMA or PA instead of MSRT and to maximize instead of minimize.

There is a difficulty in extending this method to problems in which the item costs are arbitrary values. The method is very effective when the costs are all integer and can be scaled so that the smallest cost is 1. If arbitrary costs are allowed, the algorithm can become ineffective for all  $B$  except small values of  $B$ . Consider for example the costs of \$1.00, \$1.21, \$1.27 for three items. Let the budget be \$25.00. The problem could be solved by scaling the costs to be 100, 120, and 125 and the budget to be 2500, but then too many values of  $b$  must be considered when many of them are not possible. Alternatively the algorithm can step to the "next possible value" and will consider the following sequence of values

$b = 100, 121, 127, 200, 221, 227, 242, 248, 254, 300, 321, 327, \dots$

As the process continues, depending on the relative values of the costs, the sequence becomes more dense and may eventually include all possible values of  $b$ . This is ineffective and cumbersome for large values of  $B$ .

It is also not possible to apply the modified recursion for the performance constrained problems. On the other hand, the budget constrained problems are solved very rapidly and the relationship between cost and performance can easily be determined from solving the budget constrained problem using the first recursion. In fact, if MSRT is minimized for a budget of  $B$ , the solution is actually obtained for all values of  $b$  up to and including  $B$ . Those results reveal the relationship between performance and budget.

# APPENDIX-PROGRAM LISTINGS

FILE: DPFILE EXEC A1 NAVAL POSTGRADUATE SCHOOL

FILEDEF 07 DISK (PERM  
FILEDEF 08 DISK RUSS10 DATA (PERM  
FILEDEF 06 TERM (LRECL 133 RECFM FB PERM  
FILEDEF 02 DISK (PERM

FILE: FILE FT07F001 A1 NAVAL POSTGRADUATE SCHOOL

10	1	10	NEW	0.	300.	1.	MIN	SUM	2
			300.		300.				
10			CLD						
			295.		295.				
10			CLD						
			290.		290.				
10			OLD						
			285.		285.				
10			OLD						
			280.		280.				
10			CLD						
			275.		275.				

FILE: RUSS10 DATA A1 NAVAL POSTGRADUATE SCHOOL

2 0

1.	1.	1.	1.	.0137
2.	2.	1.	1.	.0274
3.	5.	1.	1.	.0137
5.	10.	1.	1.	.0822
10.	20.	1.	1.	.0274
25.	5.	1.	1.	.0027
1.	1.	1.	1.	.0054
1.	100.	3.	1.	.0411
.5	50.	1.	1.	.0082
2.	10.	3.	1.	.1370

10 300 (OB,CON)OCJST 1SMA 2MSRT 3AVAIL - DATA ACEBD=LAM,COST,ESS,T,M2  
2 1 1 5 1 1 3 1 1  
FOR RECUR THE LAST TWO LINE MUST BE PLACED FIRST INSTEAD OF THE  
CURRENT LINE. IT IS FOR DP4



## DATA DESCRIPTION FOR CARD NUMBER 1

COLUMNS	JUSTIFY	VARIABLE NAME	MEANING
1 - 5	COL 5	NMAX	THE NUMBER OF STAGES IN THE PROBLEM
6 - 10	COL 10	MTAPE	LOGICAL TAPE NO OF THE MAIN TAPE IF THIS IS LEFT BLANK, THE COMPUTER WILL USE STANDARD SCRATCH TAPE 3
11 - 15	COL 15	SOLVE	= OLD IF THE OPTIMAL DECISION FUNCTIONS HAVE ALREADY BEEN CALCULATED AND ARE LOADED ON LOGICAL TAPE MTAPE = NEW IF THE OPTIMAL DECISION FUNCTIONS MUST BE CALCULATED AND STORED ON LOGICAL TAPE MTAPE BEFORE SOLVING THE PROBLEM

## DATA DESCRIPTION FOR THE NEXT GROUP OF CARDS

IF SOLVE = NEW ON CARD NUMBER 1, THEN EACH OF THE NMAX STAGES MUST BE DESCRIBED BY THE CARDS DISCUSSED BELOW. YOU CAN USE 1 CARD PER STAGE OR YOU CAN MAKE 1 CARD DESCRIBE MANY ADJACENT STAGES IF THEY ARE SIMILAR. THE STAGES MUST BE DESCRIBED IN NUMERICAL ORDER STARTING WITH STAGE 1 (I.E. STAGE 1, THEN 2, ..., THEN NMAX) OMIT THIS PACK OF CARDS WHICH DESCRIBE THE STAGES IF SOLVE = OLD

COLUMNS	JUSTIFY	VARIABLE NAME	MEANING
1 - 5	COL 5	NSTAGE	LOWEST NUMBERED STAGE FOR WHICH THIS CARD APPLIES
6 - 10	COL 10	NDITTO	HIGHEST NUMBERED STAGE FOR WHICH THIS CARD APPLIES. IF THESE COLUMNS ARE LEFT BLANK, THEN NDITTO WILL BE TAKEN AS EQUAL TO NSTAGE
11 - 20	ANY	XLOW	LOWEST VALUE OF XN FOR STAGE NSTAGE
21 - 30	ANY	XHIGH	HIGHEST VALUE OF XN FOR STAGE NSTAGE
31 - 40	ANY	DELX	INCREMENT IN XN FOR STAGE NSTAGE
41 - 46	COL 46	XMODE	= MIN IF STAGE NSTAGE IS TO BE MINIMIZED = MAX IF STAGE NSTAGE IS TO BE MAXIMIZED
47 - 52	COL 52	XSTAGE	= SUM IF THE COMPOSITION OPERATOR BETWEEN STAGES NSTAGE AND NSTAGE-1 IS ADDITION = MULT IF THE COMPOSITION OPERATOR BETWEEN STAGES NSTAGE AND NSTAGE-1 IS MULTIPLICATION = INMAX IF THE COMPOSITION OPERATOR BETWEEN STAGES NSTAGE AND NSTAGE-1 IS TO MINIMIZE THE MAXIMUM INDIVIDUAL STAGE RETURN

		=MAXMIN		IF THE COMPOSITION OPERATOR BETWEEN STAGES NSTAGE AND NSTAGE-1 IS TO MAXIMIZE THE MINIMUM INDIVIDUAL STAGE RETURN
		=		THIS VARIABLE IS NOT USED WHEN NSTAGE=1, THEREFORE COLUMNS 47 TO 52 ARE IGNORED WHEN NSTAGE=1 AND MAY BE LEFT BLANK HOWEVER IF NDITTO IS GREATER THAN 1, THEN XSTAGE MUST BE SPECIFIED IN ACCORDANCE WITH THE COMPOSITION OPERATOR FOR THE REST OF STAGES DESCRIBED ON THIS CARD
53 - 55	COL 55	MODES	= 1	IF THE FIBONACCI SEARCH IS TO BE USED
			= 2	IF THE FIBONACCI SEARCH IS NOT TO BE USED

DATA DESCRIPTION OF LAST CARD FOR PROBLEM  
 THE OPTIMUM DECISIONS ARE TO BE PRINTED OUT FOR A PROCESS WITH NMAX STAGES, FURTHER, FN(XN) IS TO BE OPTIMIZED WITH RESPECT TO XN WHEN THE PROGRAM COMPLETES THE REQUESTED PRINTOUT IT WILL RETURN TO THE BEGINNING AND START A NEW PROBLEM IF THERE IS DATA, IF NO DATA, IT WILL EXIT

COLUMNS	JUSTIFY	VARIABLE NAME	MEANING
1 - 15	ANY	XN1	AT STAGE NMAX, OPTIMIZE FN(XN) FOR XN NOT LESS THAN XN1
16 - 30	ANY	XN2	AT STAGE NMAX, OPTIMIZE FN(XN) FOR XN NOT GREATER THAN XN2

DESCRIPTION OF THE SUBROUTINE TRANFM  
 THE MAIN PROGRAM TRANSMITS XN, DN, AND NUMBER (THE SEQUENCE NUMBER OF THE PROBLEM) TO THE SUBROUTINE AND THE SUBROUTINE CALCULATES YN (THE OUTPUT OF THE STAGE I.E. YN=XN-1)

DESCRIPTION OF THE SUBROUTINE STGRET  
 THE MAIN PROGRAM TRANSMITS XN, DN, YN, AND NUMBER TO THE SUBROUTINE, AND THE SUBROUTINE CALCULATES RN (THE STAGE RETURN)

DESCRIPTION OF SUBROUTINE STORE  
 THE MAIN PROGRAM CALLS THE SUBROUTINE RIGHT AFTER READING THE FIRST DATA CARD AND DEFINING NUMBER. THE PURPOSE OF THIS SUBROUTINE IS TO ALLOW THE STORING OF CONSTANTS IN COMMON STORAGE FOR THE POSSIBLE USE OF SUBROUTINES STGRET AND TRANFM

DATA WHICH WILL BE READ FROM DATA CARDS AT OBJECT TIME BY THE SUBROUTINE STORE MUST BE INSERTED BETWEEN THE FIRST DATA CARD AND THE GROUP OF CARDS WHICH DESCRIBE THE INDIVIDUAL STAGES

DESCRIPTION OF SUBROUTINE DLIMIT  
 THE MAIN PROGRAM TRANSMITS N, XN, AND NUMBER TO THE SUBROUTINE,  
 AND THE SUBROUTINE CALCULATES DLJW, DHIGH, AND DELD  
 WHERE DLOW = LOWEST VALUE OF DN FOR THAT PARTICULAR VALUE OF N AND  
 XN  
 DHIGH = HIGHEST VALUE OF DN FOR THAT PARTICULAR VALUE OF N  
 AND XN  
 DELD = INCREMENT IN DN FOR THAT PARTICULAR VALUE OF N AND XN  
 IF ANY OF THESE VARIABLES DO NOT CHANGE, THEY CAN BE SET IN  
 SUBROUTINE STORE AND NOT MENTIONED IN SUBROUTINE DLIMIT

IF ANY SUBROUTINE SETS NUFF TO -1 THEN THE PROGRAM IGNORES THE  
 VALUE OF DN BEING PROCESSED AT THAT MOMENT, AND IGNORES ALL LARGER  
 VALUES OF DN FOR THAT PARTICULAR VALUE OF XN AT THAT PARTICULAR  
 STAGE--THIS FEATURE IS DISABLED WHEN USING THE FIBONACCI SEARCH

THIS PROGRAM USES TAPES 3 AND 4 FOR SCRATCH  
 NSS2 ZERO FOR ENTIRE PROBLEM SOLVING AND OUTPUT  
 NSS2 NON-ZERO FOR OUTPUT FROM PREVIOUS PROBLEM  
 DIMENSION FN(10001), FNM1(10001), JNOFXN(10001)  
 REAL \* 8 HOLMIN/' MIN' /, HOLOLD/' OLD' /  
 REAL \* 8 XMODE, XSTAGE, SOLVE  
 REAL \* 8 TAB(4)  
 COMMON /CON/TAB  
 COMMON XN, N, JN, YN, RN, SLOW, XHIGH, DELX, DLOW, DHIGH, DELD, NUMBER  
 COMMON YLOW, YHIGH, DELY, NUFF, FN, JTOP, JNFLAG, NMAX, XLAM  
 COMMON A, B, C, D, E, SUMELT, KODE1, KODE2  
 EQUIVALENCE (FN(1), FNM1(1), JNOFXN(1))  
 WRITE(6,1000) HOLMIN, HOLOLD, (TAB(I), I=1,4)  
 1000 FORMAT(1X, 6A6)  
 MAX=10001  
 INTAPE=7  
 NOUTPE=6  
 CALL SEARCH(0.0,0.0,-1.0,0.0,DNBEST,BEST)  
 NPAGE=0  
 NUMBER=0  
 240 READ(INTAPE,100,END=2000) NMAX,MTAPE, SOLVE  
 100 FORMAT(2I5,A5)  
 MTAP = MTAPE  
 NDITTO=0  
 NCALC=4  
 IF(MTAPE-4) 405,501,405  
 501 NCALC=3  
 GO TO 172  
 405 IF(MTAPE) 322,322,172  
 322 MTAPE=3  
 172 NTAPE=MTAPE  
 REWIND MTAPE  
 NUMBER=NUMBER+1  
 CALL STORE WAS MOVED FROM HERE  
 IF(SOLVE-HOLOLD) 301,302,301  
 302 NSS2=1  
 NPAGE=NPAGE+1  
 WRITE(NOUTPE,152) NPAGE  
 152 FORMAT(1H1100X4HPAGE14)  
 WRITE(NOUTPE,303) MTAPE  
 303 FORMAT(58H THE TABLES OF FN(XN) AND DN(XN) WHICH ARE ON LOGICAL T  
 1APE15,35H WILL BE USED TO SOLVE THIS PROBLEM)  
 READ(MTAPE) NMAX  
 REWIND MTAPE  
 GO TO 304  
 301 NSS2=0  
 CALL STORE  
 NPAGE=NPAGE+1  
 WRITE(NOUTPE,152) NPAGE  
 WRITE(NOUTPE,305) MTAPE  
 305 FORMAT(89H THE COMPUTER IS TO CALCULATE TABLES OF FN(XN) AND DN(XN

```

1), AND STORE THEM ON LOGICAL TAPE I5)
REWIND NCALC
500 WRITE(MTAPE) NMAX
304 WRITE(NOUTPE,306) NMAX
306 FORMAT(16H THE PROBLEM HAS I5,7H STAGES)
314 IF(NSS2) 197,109,197
109 LINES = 6
WRITE(NOUTPE,2006) NMAX, MTAP, SOLVE
2006 FORMAT(/// ' DATA CARD ', 2I7, A7)
121 DO 123 N=1,NMAX
NM1=N-1
IF(NDITTO-N) 241,132,132
241 READ(1NTAPE,124) NSTAGE,NDITTO,XLOW,XHIGH,DELX,XMODE,XSTAGE,MODES
124 FORMAT(2I5,3E10.6,2A6,I3)
WRITE(NOUTPE,2007) NSTAGE, NDITTO, XLOW, XHIGH, DELX, XMODE,
1 XSTAGE, MODES
2007 FORMAT(/// ' DATA CARD ', 2I5, 2X, 3F10.6, 2A6, I3)
IF(LINES-41) 30,30,31
31 NPAGE=NPAGE+1
WRITE(NOUTPE,152) NPAGE
LINES=1
30 LINES = LINES + 9
ITOP=(XHIGH-XLOW)/DELX+1.001
IF(MODES-1) 600,601,600
600 MODES=-1
601 IF(NSTAGE-N) 125,245,125
125 WRITE(NOUTPE,127)
127 FORMAT(23H1DATA CARD OUT OF ORDER)
GO TO 2003
245 IF(NSTAGE-NDITTO) 246,126,126
246 WRITE(NOUTPE,247) N,NSTAGE,NDITTO
247 FORMAT(///46H THE DESCRIPTION WHICH APPEARS BELOW FOR STAGE,I5,17H
1 APPLIES TO STAGE,I5,11H THRU STAGE,I5,10H INCLUSIVE)
LINES=LINES+4
XXMODE = XMODE - HOLMIN
WRITE(6,1001) XXMODE
1001 FORMAT(1X, A6)
126 IF(XMODE-HOLMIN) 230,231,230
231 XMODE=1.0
WRITE(NOUTPE,232) N
232 FORMAT(//6H STAGE I5,18H IS A MINIMIZATION)
GO TO 520
230 XMODE=-1.0
WRITE(NOUTPE,234) N
234 FORMAT(//6H STAGE I5,18H IS A MAXIMIZATION)
520 DO 129 I=1,4
IF(XSTAGE-TAB(I)) 129,130,129
129 CONTINUE
521 NNFLAG=5
GO TO 502
130 NNFLAG=1
502 IF(ITOP-MAX) 316,316,112
316 WRITE(NOUTPE,110) XLOW,XHIGH,DELX
110 FORMAT(63H THE STATE VARIABLE IS BEING TREATED AS DISCRETE, GOING F
1 ROM XN=E14.5,7H TO XN=E14.5,13H IN STEPS OF E14.5)
IF(MODES) 318,318,321
318 WRITE(NOUTPE,510)
510 FORMAT(41H TOTAL ENUMERATION IS USED FOR THIS STAGE)
GO TO 320
321 WRITE(NOUTPE,511)
511 FORMAT(40H FIBONACCI SEARCH IS USED FOR THIS STAGE)
320 GO TO (11,12,13,14,132),NNFLAG
11 WRITE(NOUTPE,16) NM1,N
16 FORMAT(40H THE COMPOSITION OPERATOR BETWEEN STAGES I6,4H AND I6,12H
1 IS ADDITION)
GO TO 132
12 WRITE(NOUTPE,17) NM1,N
17 FORMAT(40H THE COMPOSITION OPERATOR BETWEEN STAGES I6,4H AND I6,18H
1 IS MULTIPLICATION)
GO TO 132
13 WRITE(NOUTPE,18) NM1,N
18 FORMAT(40H THE COMPOSITION OPERATOR BETWEEN STAGES I6,4H AND I6,34H

```



```

11S TO MAXIMIZE THE MINIMUM RETURN)
GO TO 132
14 WRITE(NOUTPE,19) NM1,N
19 FORMAT(40H THE COMPOSITION OPERATOR BETWEEN STAGES I6,4H AND I6,34H
11S TO MINIMIZE THE MAXIMUM RETURN)
132 DO 136 I=1,ITOP
134 BEST=1.0E+35*XMODE
X=I-1
XN=XLOW+X*DELX
NUFF=1
CALL DLIMIT
IF(MODES) 602,602,603
603 CALL SEARCH(DLOW,DHIGH,DELD,XMODE,DNBEST,BEST)
GO TO 400
602 KTOP=(DHIGH-DLOW)/DELD+1.001
223 DO 137 J=1,KTOP
X=J-1
DN=DLOW+X*DELD
JUMP MAY BE SET TO 1 IN TRANFM TO INDICATE AN INFEASIBLE XN-1
C
C JUMP=0
CALL TRANFM
CALL STGRET
XMODE=1 FOR MIN, -1 FOR MAX
C
C IF(JUMP.EQ.1) RN=999999*XMODE
IF(NUFF) 400,401,401
401 IF(N-1) 504,504,503
504 QN=RN
GO TO 143
503 K=(YN-YLOW)/DELY+1.001
IF(K) 212,212,209
212 WRITE(NOUTPE,210) N,XN,DN,YN
210 FORMAT(///9H AT STAGE I5,9H WITH XN=E15.8,8H AND DN=E15.8,12H XN-1
1EQUALSE16.8,21H AND IS OUT OF LIMITS///)
GO TO 2003
209 IF(K-JTOP) 205,206,212
206 RNMI=FNM1(JTOP)
GO TO 207
205 X=K-1
X=YLOW+X*DELY
RNMI=FNM1(K)+((FNM1(K+1)-FNM1(K))* (YN-X)/DELY
207 GO TO (21,22,23,24,248),NNFLAG
248 WRITE(NOUTPE,249) N
249 FORMAT(43H1COMPOSITION OPERATOR NOT DEFINED FOR STAGE,I5)
GO TO 2003
21 QN=RN+RNMI
GO TO 143
22 QN=RN-RNMI
GO TO 143
23 IF(RN-RNMI) 141,141,142
141 QN=RN
GO TO 143
142 QN=RNMI
GO TO 143
24 IF(RN-RNMI) 145,145,146
145 QN=RNMI
GO TO 143
146 QN=RN
143 IF((QN-BEST)*XMODE) 144,137,137
144 BEST=QN
DNBEST=DN
137 CONTINUE
400 WRITE(NCALC) BEST,DNBEST
136 CONTINUE
REWIND NCALC
WRITE(NTAPE) XLOW,XHIGH,DELX,ITOP,XMODE
DO 404 I=1,ITOP
404 READ(NCALC) DUM,DNOFXN(I)
REWIND NCALC
WRITE(NTAPE) (DNOFXN(II),II=1,ITOP)
DO 402 I=1,ITOP
402 READ(NCALC) FN(I),DUM
REWIND NCALC

```

```

YLOW=XLOW
YHIGH=XHIGH
DELY=DELX
JTOP=ITOP
123 CONTINUE
WRITE(MTAPE) (FN(II),II=1,ITOP)
161 END FILE MTAPE
197 REWIND MTAPE
READ(MTAPE) NMAX
N=NMAX
J=N-1
IF(J) 407,407,201
201 DO 522 I=1,J
READ(MTAPE)
522 READ(MTAPE)
407 READ(MTAPE) XLOW,XHIGH,DELX,ITOP,XMODE
READ(MTAPE)
READ(MTAPE) (FN(I),I=1,ITOP)
BACKSPACE MTAPE
BACKSPACE MTAPE
READ(MTAPE,162) XN1,XN2
162 FORMAT(2E15.8)
NPAGE=NPAGE+1
WRITE(NOUTPE,152) NPAGE
WRITE(NOUTPE,2008) XN1, XN2
2008 FORMAT(/// ' DATA CARD ', 2F20.3 ///)
IF(XMODE) 323,323,324
324 WRITE(NOUTPE,325) N
325 FORMAT(29H THE PROBLEM IS TO MINIMIZE AI6,14H STAGE PROCESS)
GO TO 327
323 WRITE(NOUTPE,326) N
326 FORMAT(29H THE PROBLEM IS TO MAXIMIZE AI6,14H STAGE PROCESS)
327 WRITE(NOUTPE,328) XN1,XN2
328 FORMAT(42H XN IS TO BE CHOSEN OPTIMALLY BETWEEN XN=1PE14.5,8H AN
1D XN=1PE14.5)
171 IF(XLOW-XN1) 175,175,176
176 WRITE(NOUTPE,177) XLOW,XHIGH
177 FORMAT(47H THE PROGRAM ONLY HAS INFORMATION ON XN BETWEEN E16.8,4H
1A DE16.8)
GO TO 240
175 IF(XN2-XHIGH) 181,181,176
181 IF(XN1-XN2) 178,178,176
178 IX1=(XN1-XLOW)/DELX+1.001
IX2=(XN2-XLOW)/DELX+1.001
BEST=1.0E+35*XMODE
DO 182 J=IX1,IX2
IF((FN(J)-BEST)*XMODE) 183,182,182
183 BEST=FN(J)
JSAVE=J
182 CONTINUE
X=JSAVE-1
XN=XLOW+X*DELX
WRITE(NOUTPE,184) XN,BEST
184 FORMAT(12H OPTIMAL XN=1PE14.5,16H OPTIMAL RETURN=1PE14.5)
READ(MTAPE) (DNOFXN(II),II=1,ITOP)
186 DN=DNOFXN(JSAVE)
CALL TRANFM
204 WRITE(NOUTPE,188)
188 FORMAT(7H0 N18X2HXN18X2HDN15X4HXN-1)
NN=0
203 WRITE(NOUTPE,189) N,XN,DN,YN
189 FORMAT(I7,1P3E20.5)
NN=NN+1
N=N-1
IF(N-1) 244,193,193
244 GO TO 240
193 DO 406 I=1,4
406 BACKSPACE MTAPE
READ(MTAPE) XLOW,XHIGH,DELX,ITOP,XMODE
READ(MTAPE) (DNOFXN(II),II=1,ITOP)
L=(YN-XLOW)/DELX+1.001
IF(L) 214,214,215

```

```

214 L=1
219 NP1=N+1
WRITE(NOUTPE,210) NP1,XN,DN,YN
NN=NN+7
215 IF(L-ITOP) 218,216,217
216 DN=DNOFXN(L)
XN=YN
GO TO 196
217 L=ITOP
GO TO 219
218 XN=YN
194 X=L-1
X=XLOW+X*DELX
DN=DNOFXN(L)+(DNOFXN(L+1)-DNOFXN(L))*(XN-X)/DELX
196 CALL TRANFM
IF(NN-50) 203,203,220
220 NPAGE=NPAGE+1
WRITE(NOUTPE,152) NPAGE
GO TO 204
112 WRITE(NOUTPE,113) MAX,ITOP
113 FORMAT(72H THIS PROGRAM LIMITS THE NUMBER OF DISCRETE STEPS OF THE
1 STATE VARIABLE TO 16,26H AND THIS PROBLEM REQUIRES 16)
2002 FORMAT('1',' ERROR HALT')
2003 WRITE(6,2002)
STOP
2000 WRITE(6,2001)
2001 FORMAT('1',' END OF DATA FILE')
STOP
END
CSEARCH DISCRETE FIBONACCI SEARCH SUBROUTINE
SUBROUTINE SEARCH(AA,BB,DELYY,XXMODE,YYBEST,BEST)
DIMENSION F(150)

OPTIMIZE WITH RESPECT TO Y BETWEEN AA AND BB IN STEPS OF DELYY
STORE OPTIMUM Y IN YYBEST AND THE OPTIMUM VALUE OF THE OBJECTIVE
FUNCTION IN BEST
XXMODE=-1 FOR MAXIMIZE
XXMODE=1 FOR MINIMIZE
SUBROUTINE MUST BE INITIALIZED BY CALLING IT WITH DELYY=-1.0 AT
LEAST ONCE BEFORE IT IS USED FOR A SEARCH
DELY=DELYY
A=AA
B=BB
XMODE=XXMODE
IF(DELY) 100,100,101
100 F(1)=1.0
F(2)=2.0
DO 1 I=3,150
F(I)=F(I-1)+F(I-2)+1.0
IF(F(I)-1.0E+35) 1,2,2
2 II=I
RETURN
1 CONTINUE
II=150
RETURN
101 YHI=B
YLO=A
FNO=(YHI-YLO)/DELY+1.0
DO 5 N=1,II
IF(FNO-F(N)) 6,6,5
5 CONTINUE
WRITE(6,7)
7 FORMAT(38H1ERROR, TOO MANY POINTS TO BE SEARCHED)
2002 FORMAT('1',' ERROR HALT')
2003 WRITE(6,2002)
STOP
6 IF(N-2) 42,41,40
40 Y1=F(N-2)*DELY+YLO
CALL FUNCTN(Y1,GY1)
Y2=F(N-1)*DELY+YLO
CALL FUNCTN(Y2,GY2)
15 N=N-1

```

```

      IF(N-1) 102,102,103
103  IF((GY2-GY1)*XMODE) 19,19,20
      YLO=Y1+DELY
      Y1=Y2
      GY1=GY2
      Y2=F(N-1)*DELY+YLO
      IF(Y2-B) 104,104,105
105  GY2=1.0E+35*XMODE
      GO TO 16
104  CALL FUNCTN(Y2,GY2)
      GO TO 16
      20 YHI=Y2-DELY
      Y2=Y1
      GY2=GY1
      IF(N-2) 34,34,35
34  FNM2=0.0
      GO TO 36
35  FNM2=F(N-2)
36  Y1=FNM2*DELY+YLO
      CALL FUNCTN(Y1,GY1)
      GO TO 16
102  IF((GY2-GY1)*XMODE) 110,111,111
110  YYBEST=Y2
      BEST=GY2
      RETURN
111  YYBEST=Y1
      BEST=GY1
      RETURN
41  Y1=A
      CALL FUNCTN(Y1,GY1)
      Y2=B
      CALL FUNCTN(Y2,GY2)
      GO TO 102
42  Y1=A
      CALL FUNCTN(Y1,GY1)
      GO TO 111
      END
CFUNCTN
      SUBROUTINE FUNCTN(Y,GY)
      DIMENSION FN(10001),FNM1(10001),DNOFXN(10001)
      REAL * 8 TAB(4)
      COMMON /CON/TAB
      COMMON XN,N,DN,YN,RN,SLOW,XHIGH,DELX,DLOW,DHIGH,DELD,NUMBER
      COMMON YLOW,YHIGH,DELY,NUFF,FN,JTOP,NNFLAG,NMAX,XLAM
      COMMON A,B,C,D,E,SUMELT,KODE
      EQUIVALENCE (FN(1),FNM1(1),DNOFXN(1))
      DN=Y
      NOUTPE=6
      C JUMP MAY BE SET TO 1 IN TRANFM TO INDICATE AN INFEASIBLE XN-1
      C JUMP=0
      CALL TRANFM
      CALL STGRET
      C XMODE=1 FOR MIN, -1 FOR MAX
      C IF(JUMP.EQ.1)RN=999999*XMODE
      IF(N-1) 146,146,401
401  K=(YN-YLOW)/DELY+1.001
      IF(K) 212,212,209
212  WRITE(NOUTPE,210) N,XN,DN,YN
210  FORMAT(///9H AT STAGE I5,9H WITH XN=E15.8,8H AND DN=E15.8,12H XN-1
1  EQUALSE16.8,21H AND IS OUT OF LIMITS//)
2002  FORMAT('1',' ERROR HALT')
2003  WRITE(NOUTPE,2002)
      STOP
209  IF(K-JTOP) 205,206,212
206  RNM1=FNM1(JTOP)
      GO TO 207
205  X=K-1
      X=YLOW+X*DELY
      RNM1=FNM1(K)+(FNM1(K+1)-FNM1(K))*(YN-X)/DELY
207  GO TO (21,22,23,24),NNFLAG
21  QN=RN+RNM1
      GO TO 137

```



```
22 QN=RN*RNM1
   GO TO 137
23 IF(RN-RNM1) 141,141,142
141 QN=RN
   GO TO 137
142 QN=RNM1
   GO TO 137
24 IF(RN-RNM1) 145,145,146
145 QN=RNM1
   GO TO 137
146 QN=RN
137 GY=QN
   RETURN
   END
```

CSTORE

```

SUBROUTINE STORE
DIMENSION FN(10001),FNM1(10001),NOFXN(10001),A(101),B(101),C(101)
DIMENSION D(101),E(101)
REAL * 8 TAB(4)
COMMON /CON/TAB
COMMON XN,N,DN,YN,RN,SLOW,XHIGH,DELX,DLOW,DHIGH,DELD,NUMBER
COMMON YLOW,YHIGH,DELY,NUFF,FN,JTOP,VNFLAG,NMAX,XLAM
COMMON A,B,C,D,E,SUMELT,KODE1,KODE2
EQUIVALENCE (FN(1),FNM1(1),DNOFXN(1))
INDATA=8
NOUTPE=6
READ (INDATA,808) KODE1,KODE2
808 FORMAT (2I3)
IF(KODE1.EQ.0) WRITE(NOUTPE,110)
IF(KODE1.EQ.1) WRITE(NOUTPE,101)
IF(KODE1.EQ.2) WRITE(NOUTPE,202)
IF(KODE1.EQ.3) WRITE(NOUTPE,303)
110 FORMAT (30H THE OBJECTIVE IS COST )
101 FORMAT (30H THE OBJECTIVE IS SMA )
202 FORMAT (30H THE OBJECTIVE IS MSRT )
303 FORMAT (30H THE OBJECTIVE IS AVAILABILITY)
IF(KODE2.EQ.0) WRITE(NOUTPE,900)
IF(KODE2.EQ.1) WRITE(NOUTPE,901)
IF(KODE2.EQ.2) WRITE(NOUTPE,902)
IF(KODE2.EQ.3) WRITE(NOUTPE,903)
900 FORMAT (30H CONSTRAINT ON COST )
901 FORMAT (30H CONSTRAINT ON SMA )
902 FORMAT (30H CONSTRAINT ON MSRT )
903 FORMAT (30H CONSTRAINT ON AVAILABILITY )
DO 333 K=1,NMAX
READ (INDATA,999) A(K),C(K),E(K),B(K),D(K)
999 FORMAT (5F10.4)
WRITE(NOUTPE,999) A(K),B(K),C(K),D(K),E(K)
333 CONTINUE
SUMELT=0.
DO 30 I=1,NMAX
30 SUMELT=SUMELT+E(I)*A(I)*B(I)
WRITE(6,765)SUMELT
765 FORMAT(F10.3)
RETURN
END

```

```

STGRET
SUBROUTINE STGRET
DIMENSION FN(10001),FNM1(10001),DNOFXN(10001),A(101),B(101),C(101)
DIMENSION D(101),E(101)
REAL * 8 TAB(4)
COMMON /CON/TAB
COMMON XN,N,DN,YN,RN,SLOW,XHIGH,DELX,DLOW,DHIGH,DELD,NUMBER
COMMON YLOW,YHIGH,DELY,NUFF,FN,JTOP,NFLAG,NMAX,XLAM
COMMON A,B,C,D,E,SUMELT,KODE1,KODE2
EQUIVALENCE (FN(1),FNM1(1),DNOFXN(1))
IF(KODE1.EQ.0)GO TO 77
IDN=INT(DN)
AB=A(N)*B(N)
IS=IDN
TERM=0.
TEMP=0.
IF(IS.LT.0) GO TO 11
TERM=EXP(-AB)
TEMP=TERM
IF(IS.EQ.0) GO TO 11
DO 10 I=1,IS
TEMP=TEMP*AB/I
IF(TERM.GE..99999) GO TO 11
IF(TEMP.LE..00001) GO TO 11
10 TERM=TERM +TEMP
11 CDF=TERM
20 CONTINUE
IF(KODE1.NE.1)GO TO 40
C FOLLOWING CHANGED 120183
C SMA=(AB*(1.-TEMP)+(IDN-AB)*(1.-CDF))/AB
SMA=(1.-TEMP)+(IDN-AB)*(1.-CDF)/AB
RN=E(N)*AB*SMA/SUMELT
RETURN
40 TWUS=(1.-CDF)*(AB*AB-2.*AB*IDN+IDN*(IDN+1))/(2.*A(N))
X+TEMP*B(N)*(AB-IDN)/2.
IF(KODE1.EQ.3)GO TO 50
RN=E(N)*TWUS/SUMELT
RETURN
50 CONTINUE
AMSR=TWUS/AB
AMTBF=1./A(N)
AMTTR=D(N)
RN=(AMTBF)/(AMTBF+AMTTR+AMSR)
RETURN
77 RN=C(N)*DN
C IF(KODE2.EQ.1.AND.N.EQ.1.AND.YN.LT.XN)RN=999999
RETURN
END

```

CDLIMIT

```

SUBROUTINE DLIMIT
  DIMENSION FN(10001),FNM1(10001),DNOFXN(10001),A(101),B(101),C(101)
  DIMENSION D(101),E(101)
  REAL * 8 TAB(4)
  COMMON /CON/TAB
  COMMON XN,N,YN,RN,SLOW,XHIGH,DELX,DLOW,DHIGH,DELD,NUMBER
  COMMON YLOW,YHIGH,DELY,NUFF,FN,JTOP,NFLAG,NMAX,XLAM
  COMMON A,B,C,D,E,SUMELT,KODE1,KODE2
  EQUIVALENCE (FN(1),FNM1(1),DNOFXN(1))
  IF(KODE2.NE.0)GO TO 37
  DLOW=0.
  DELD=1.
  DHIGH=AMINI(XN/C(N),50.)
  RETURN
37 AB=A(N)*B(N)
  DO 555 ID=1,30
  IS=ID-1
  TERM=EXP(-AB)
  TEMP=TERM
  IF(IS.EQ.0) GO TO 11
  DO 10 I=1,IS
  TEMP=TEMP*AB/I
  IF(TEMP.GE..99999) GO TO 11
  IF(TEMP.LE..00001) GO TO 11
C 10 TERM=TERM +TEMP
  11 CDF=TERM
  40 TWUS=(1.-CDF)*(AB*AB-2.*AB*IS+IS*(IS+1))/(2.*A(N))
  X+TEMP*B(N)*(AB-IS)/2.
  IF(KODE2.EQ.2.AND.E(N)*TWUS/SUMELT.LE.XN)GO TO 556
C CHANGED FOLLOWING TO 555 FROM 556
  IF(KODE2.EQ.2)GO TO 555
  IF(KODE2.EQ.3)GO TO 554
  SSS=(1.-TEMP)+(IS-AB)*(1.-CDF)/AB
  RR=E(N)*AB*SSS/SUMELT
  Y=XN-RR
  DLOW=0.
C THE NUMBER IN THE NEXT LINE HAS A BIG EFFECT. IT SHOULD NOT.
  IF(N.EQ.1)DLOW=1000.
  IF(N.EQ.1.AND.Y.GT.0..AND.ID.LT.30)GO TO 555
  IF(N.EQ.1.AND.Y.LE.0.)DLOW=IS
136 DHIGH=30.+DLOW
  IF(DLOW.EQ.1000.) DHIGH=1000.
  DELD=1.
  RETURN
554 AMSRT=TWUS/AB
  AMTBF=1./A(N)
  AMTTR=D(N)
  AV=AMTBF/(AMTBF+AMTTR+AMSRT)
  IF(AV.GE.XN) GO TO 556
555 CONTINUE
  IS=30.
556 DLOW=IS
  DELD=1.
  DHIGH=50.
  RETURN
END
BLOCK DATA
COMMON /CON/TAB
REAL * 8 TAB(4) / ' SUM', ' MULT','MAXMIN','MINMAX'/
END

```

TRANFM

```

SUBROUTINE TRANFM
  DIMENSION FN(10001),FNM1(10001),DNOFXN(10001),A(101),B(101),C(101)
  DIMENSION D(101),E(101)
  REAL * 8 TAB(4)
  COMMON /CON/TAB
  COMMON XN,N,DN,YN,RN,SLOW,XHIGH,DELX,DLOW,DHIGH,DELD,NUMBER
  COMMON YLOW,YHIGH,DELY,NUFF,FN,JTOP,VNFLAG,VMAX,XLAM
  COMMON A,B,C,D,E,SUMELT,KODE1,KODE2
  EQUIVALENCE (FN(1),FNM1(1),DNOFXN(1))
  IF(KODE2.NE.0)GO TO 12
  YN=XN-C(N)*DN
  RETURN
12 IDN=INT(DN)
  AB=A(N)*B(N)
  IS=IDN
  TERM=0.
  TEMP=0.
  IF(IS.LT.0) GO TO 11
  TERM=EXP(-AB)
  TEMP=TERM
  IF(IS.EQ.0) GO TO 11
  DO 10 I=1,IS
    TEMP=TEMP*AB/I
  IF(TERM.GE..99999) GO TO 11
  IF(TEMP.LE..00001) GO TO 11
10 TERM=TERM+TEMP
11 CDF=TERM
40 TWUS=(1.-CDF)*(AB*AB-2.*AB*IDN+IDN*(IDN+1))/(2.*A(N))
  X+TEMP*B(N)*(AB-IDN)/2.
  IF(KODE2.EQ.2) GO TO 50
  IF(KODE2.EQ.3) GO TO 51
  SMA=(1.-TEMP)+(IDN-AB)*(1.-CDF)/AB
  YN=XN-E(N)*AB*SMA/SUMELT

  IF(YN.LT.0) YN=0.0

  IF(N.NE.1.AND.YN.LT.0.OR.N.NE.1.AND.YN.GT.1.)JUMP=1
  IF(N.EQ.1.AND.YN.GT.0)JUMP=1
  IF(JUMP.EQ.1)YN=0.
  RETURN
51 AMSRT=TWUS/AB
  AMTBF=1./A(N)
  AMTTR=D(N)
  AV=(AMTBF)/(AMTBF+AMTTR+AMSRT)
  YN=XN/AV
  AV IS AVAIL
  RETURN
50 YN=XN-E(N)*TWUS/SUMELT
  IF(YN.LE.0.)YN=0.
  RETURN
END

```

```

      DIMENSION A(20),B(20),C(20),D(20),E(20),MINQ(20)
      DIMENSION T(10,30),F(1200),DX(10,1200),TN(10,31)
      INTEGER BUDGET,BB,BBP1,DX
      REAL*8 TIME,CTIME
      INDATA=8
      NOUTPE=6
      READ(INDATA,888) NMAX,BUDGET
888  FORMAT(2I5)
      READ(INDATA,777) (MINQ(I),I=1,NMAX)
777  FORMAT(20I5)
      DO 333 K=1,NMAX
      READ(INDATA,999) A(K),C(K),E(K),B(K),D(K)
999  FORMAT(5F10.4)
C 333  WRITE(NCUTPE,999) A(K),B(K),C(K),D(K),E(K)
      CONTINUE
      TIME=CTIME(1)
      SUMELT=0.
      DO 30 I=1,NMAX
30  SUMELT=SUMELT+E(I)*A(I)*B(I)

      DO 10 I=1,NMAX
      DX(I,1)=0
      DO 10 KP1=1,31
      K=KP1-1
      AB=A(I)*B(I)
      TERM=EXP(-AB)
      TEMP=TERM
      IF(K.EQ.0) GO TO 11
      DO 14 JI=1,K
      TEMP=TEMP*AB/JI
C 14  IF(TERM.GE..9999999) GO TO 11
C 11  IF(TEMP.LE..0000001) GO TO 11
      TERM=TERM+TEMP
      11  CDF=TERM

      40  TWUS=(1.-CDF)*(AB*AB-2.*AB*K+K*(K+1))/(2.*A(I))
      X+TEMP*B(I)*(AB-K)/2.
      110  TN(I,KP1)=E(I)*TWUS/SUMELT
      DO 70 K=1,30
      70  T(I,K)=TN(I,K+1)-TN(I,K)
      10  CONTINUE

C 456  DO 456 IJK=1,10
C 555  WRITE(6,555) (TN(KI,IJK),KI=1,NMAX)
C 555  FORMAT(5F12.7)

      DO 80 I=1,NMAX
      LIM=30/MINQ(I)
      DO 85 K=1,LIM
      TMOD=0.
      MM=MINQ(I)
      DO 90 J=1,MM
90  TMOD=TMOD+T(I,(K-1)*MINQ(I)+J)
      T(I,K)=TMOD
      85  CONTINUE
      C(I)=MINQ(I)*C(I)
      80  CONTINUE

      IBD=BUDGET+1
      DO 100 BBP1=1,IBD
      BB=BBP1-1
      XMIN=0.
      ISTAR=0
      TRY=0.
      DO 210 I=1,NMAX
      IF(BB-C(I).LT.0) GO TO 210
      IF(BB-C(I).GT.0) GO TO 211
      TRY=AMIN1(TRY,T(I,1))
      GO TO 212

```

```

211 TRY=T(I,DX(I,BBP1-C(I))+1)+F(BBP1-C(I))
212 IF (TRY.GT.XMIN) GO TO 210
   XMIN=TRY
   ISTAR=I
210 CONTINUE
   F(BBP1)=XMIN

19 DO 21 I=1,NMAX
   IF (ISTAR.EQ.0.OR.BB-C(ISTAR).LE.0) DX(I,BBP1)=0
   IF (ISTAR.EQ.0.OR.BB-C(ISTAR).LE.0) GO TO 21
   DX(I,BBP1)=DX(I,BBP1-C(ISTAR))
21 CONTINUE
   IF(ISTAR.EQ.0)GO TO 100
   IF(BBP1-C(ISTAR).LE.1) DX(ISTAR,BB+1)=1
   IF(BBP1-C(ISTAR).LE.1) GO TO 100
   DX(ISTAR,BBP1)=AMINO(DX(ISTAR,BBP1-C(ISTAR))+1,30)
100 CONTINUE
   KLOW=IBD-5
   KHIGH=IBD
   DO 101 KBD=KLOW,KHIGH
   TIME=TIME-CTIME(1)
C   WRITE(6,3456)TIME
3456 FORMAT('TIME=',F12.9)
   WRITE(6,1000)(DX(I,KBD),I=1,NMAX)
   WRITE(6,1000)(MINQ(I),I=1,NMAX)
1000 FORMAT(10I5)
   AMSRT=0.
   DO 88 I=1,NMAX
   88 AMSRT=AMSRT+IN(I,MINQ(I)*DX(I,KBD)+1)
   KBDM1=KBD-1
   WRITE (6,2000) AMSRT,KBDM1
2000 FORMAT(1X,'MSRT= ',F10.8,'    BJDGET= ',I7)
101 CONTINUE
   RETURN
   END
   FUNCTION CTIME(I)
   REAL*8 CTIME
   DATA IFL/0/
   IF( IFL.NE.0 ) GO TO 10
   IFL = 1
   CTIME = 0.000
   CALL SETIME
   RETURN
10 CALL GETIME(ITIME)
   CTIME = -ITIME*0.00002600
   RETURN
   END

```

## References

1. Dreyfus, S. E., and Law, A. M., "The Art and Theory of Dynamic Programming," Academic Press, New York, 1977.
2. Richards, F. R., and McMaster, A. W., "Wholesale Provisioning Models: Model Development," Naval Postgraduate School Technical Report NPS55-83-026, September 1983.
3. Kang, Sung Jin, "Analysis of Inventory Models with Budget Constraints" Masters Thesis, Naval Postgraduate School, September 1983.



# DISTRIBUTION LIST

	No. of Copies
Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
Defense Logistics Studies Information Exchange U. S. Army Logistics Management Center Fort Lee, VA 23801	1
Library, Code 0142 Naval Postgraduate School Monterey, CA 93943	2
Library, Code 55 Department of Operations Research Naval Postgraduate School Monterey, CA 93943	1
Associate Professor G. T. Howard Code 55 Hk, Department of Operations Research Naval Postgraduate School Monterey, CA 93943	5
Associate Professor F. Russell Richards Code 55Rh, Department of Operations Research Naval Postgraduate School Monterey, CA 93943	5
Associate Professor Alan W. McMasters, Code 54Mg Department of Administrative Sciences Naval Postgraduate School Monterey, CA 93943	5
Commanding Officer Navy Fleet Material Support Office Attn: Code 93 Mechanicsburg, PA 17055	5
Commanding Officer Naval Supply Systems Command Attn: SUP 04A Washington, DC 20376	5
Commanding Officer Navy Ships Parts Control Center Attn: Code 0412 Mechanicsburg, PA 17055	5
Commanding Officer Navy Aviation Supply Office Attn: Code SDB4-A Philadelphia, PA 19111	5





DUDLEY KNOX LIBRARY



3 2768 00347415 6